HI Masses Detected

HI Mass of a Detection

In section 4.1 of Giovanelli et al. 2005[1] (α.I), Scaling Relations, the HI mass of an optically HI source at a distance D_{Mpc} is given as

$$\frac{M_{HI}}{M_{\odot}} = 2.356\times10^5 D_{Mpc}^2 \int S(V)dV \qquad \alpha.\text{I (1)}$$

where S(V) is the HI line profile in Janskys and V is the Doppler velocity in km/s. Taking

$$\int S(V)dV \approx S_{peak}W_{km/s}$$

the integral as the peak of the flux, S_{peak}, times the velocity width, $W_{km/s}$, makes this

$$\frac{M_{HI}}{M_{\odot}} = 2.356\times10^5 D_{Mpc}^2 S_{peak}W_{km/s}. \qquad \alpha\text{I (2)}$$

Noise and Signal-to-Noise Ratio (S/N)[2]

It can be shown that the radiometer equation[3] gives

$$S_{\nu,\ rms} = \frac{2kT_{sys}}{A_e\sqrt{\tau\Delta\nu}} \qquad (1)$$

Defining the system gain, $G = Ae/2k\sqrt{(f_t)}$, the channel width as Δf_{ch}, and the integration time as $2f_t$ (accounting for the addition of the data in the two polarizations) gives Equation 3 from α.I:

$$S_{rms} = \frac{T_{sys}/G}{\sqrt{2\Delta f_{ch}t_s f_t}}$$

The factor f_t accounts for smoothing, on-off switching and other observational details. The most important of these for LBW observations is the smoothing so replace f_t with f_{smo}:

$$S_{rms} = \frac{T_{sys}/G}{\sqrt{2\Delta f_{ch}t_s f_{smo}}} \qquad (2)$$

Solving α.I (s) for S_{peak} gives

$$S_{peak} = \frac{M_{HI}}{M_{\odot}}\frac{1}{2.356\times10^5 D_{Mpc}^2 W_{km/s}}. \qquad (3)$$

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[1] Giovanelli, R., Haynes, M. P., Kent, B. R., et al. 2005, AJ, 130, 2598

[2] The following sections could not have been written without the patient assistance of Dr. Michael Jones!

[3] O'Donoghue, A. A., personal notes, "Radiometer Equation" available at uat_apps

The signal-to-noise ratio (S/N) is thus,

$$\frac{S}{N}=\frac{\frac{1}{2}S_{peak}}{S_{rms}}=\left(\frac{M_{HI}}{M_\odot}\frac{1}{2.356\times10^5 D_{Mpc}^2 W_{km/s}}\right)\left(\frac{\sqrt{2\Delta f_{ch} t_s f_{smo}}}{2\left(T_{sys}/G\right)}\right)$$

$$\frac{S}{N}=\frac{M_{HI}}{M_\odot}\frac{\sqrt{\Delta f_{ch} t_s f_{smo}}}{3.33\times10^5 D_{Mpc}^2 W_{km/s}\left(T_{sys}/G\right)}. \quad (4)$$

where the ½ in the numerator is due to the fact that with on-off switching, we observe the noise for the full time, τ_s but the source for only half the time.

SMOOTHING AND S/N

The S/N is maximized by smoothing because the noise increases as $\sqrt{f_{smo}}$, (where f_{smo} is the number of Δf_{ch} smoothed) while the source peak will increase as f_{smo}. This assumes that each smoothed channel adds signal. Smoothing past half the width of the emission starts bringing in channels with no signal so it no longer improves S/N. To reflect this, take the smoothing factor as half the width divided by the width of each channel.

$$f_{smo}=\frac{W_{km/s}/2}{\Delta v_{ch}} \quad (5)$$

To get the channel width in velocity units, use the redshift, v = cz,

$$\frac{\Delta v_{ch}}{c_{km/s}}\simeq\frac{\Delta f_{ch}}{f_0} \quad (6)$$

where f_0 is the rest frequency of the HI line, f_0 = 1420 MHz. Thus, the smoothing factor

$$f_{smo}=\frac{W_{km/s}}{2c_{km/s}}\frac{f_0}{\Delta f_{ch}} \quad (7)$$

and the signal-to-noise can be written as

$$\frac{S}{N}=\frac{M_{HI}}{M_\odot}\frac{\sqrt{\cancel{\Delta f_{ch}} t_s\left(W_{km/s} f_0/2c_{km/s}\cancel{\Delta f_{ch}}\right)}}{3.33\times10^5 D_{Mpc}^2 W_{km/s}\left(T_{sys}/G\right)}.$$

$$\frac{S}{N}=\frac{M_{HI}}{M_\odot}\frac{\sqrt{t_s\left(W_{km/s} f_0/c_{km/s}\right)}}{4.71\times10^5 D_{Mpc}^2 W_{km/s}\left(T_{sys}/G\right)}.$$

$$\boxed{\frac{S}{N}=\frac{M_{HI}}{M_\odot}\frac{\sqrt{t_s f_0}}{4.71\times10^5 D_{Mpc}^2\left(T_{sys}/G\right)\sqrt{W_{km/s} c_{km/s}}}.} \quad (8)$$

S/N for UAT Projects

According to *An Astronomer's Guide to the Arecibo 305-m Telescope*[4],

Receiver Designation	Freq Range (GHz)	System Temp[a] (K)	Gain[a] K/Jy	SEFD[a,b] Jy (at zenith)	HPBW[a,f] Az × ZA (Arcmin)
Carriage House					
lbw	1.120 – 1.730	25	10.5	2.4	3.1 × 3.5
ALFA					
Center Pix	1.225 – 1.525	30	11	2.8	3.3 × 3.7
Outer Pixs	1.225 – 1.525	30	8.5	3.5	3.3 × 3.7

a) T_{sys}, Gain and SEFD all vary with zenith angle (and to a lesser degree with azimuth), T_{sys}, and SEFD increase with zenith angle, while Gain decreases. The HPBW in ZA increases with zenith angle.
b) SEFD, the System Equivalent Flux Density (= T_{sys}/G) is the system temperature expressed in Jy/beam.

Fall LBW Observations:

For the Fall LBW observations (A2707, A2811 and A2899), T/G = SEFD = 2.4 Jy and t_s = 360 s, this gives

$$\left(\frac{S}{N}\right)_{\text{LBW, 3 min}} = \frac{M_{HI}}{M_\odot}\frac{\sqrt{(360)(1420\times10^6)}}{4.71\times10^5 D_{Mpc}^2(2.4)\sqrt{W_{km/s}(3\times10^5)}}.$$

$$\left(\frac{S}{N}\right)_{\text{LBW, 3 min}} = 1.16\times10^{-3}\frac{M_{HI}}{M_\odot}\frac{1}{D_{Mpc}^2\sqrt{W_{km/s}}}. \tag{9}$$

Giovanelli et al. argue in α.I that the spectral smoothing increases the S/N up to a maximum of $W_{km/s}$ = 200. They then substitute

$$\frac{1}{\sqrt{W_{km/s}}} = \frac{1}{\sqrt{200}}\left(\frac{200}{W_{km/s}}\right)^{-\gamma}$$

where γ = -1/2 for $W_{km/s} < 200$ and γ = -1 for $W_{km/s} \geq 200$. With the LBW galaxies likely to have $W_{km/s} < 200$, this now gives a S/N equation similar to α.I (4) (that uses ALFALFA parameters),

$$\left(\frac{S}{N}\right)_{\text{LBW, 3 min}} = 8.23\times10^{-5}\frac{M_{HI}}{M_\odot}\frac{1}{D_{Mpc}^2}\sqrt{\frac{200}{W_{km/s}}}. \tag{10}$$

Inverting equation (10) will then allow the calculation of the minimum HI mass detectable for a given S/N, distance and width,

$$\left(\frac{M_{HI}}{M_\odot}\right)_{min} = 1.22\times10^4 D_{Mpc}^2\sqrt{\frac{W_{km/s}}{200}}\left(\frac{S}{N}\right)_{\text{LBW, 3 min}} \tag{11}$$

[4] Salter, Chris, 2012, www.naic.edu/~astro/guide/

Since in the fall LBW observations, we observed ALFALFA Code 3 sources within 1000 km/s, take

$$D_{Mpc} = \frac{v}{H_0} = \frac{1000\ \mathrm{km/s}}{70\ \mathrm{km/s/Mpc}} = 14.3\ \mathrm{Mpc}.$$

For the fainter ALFALA objects, 4.5 < S/N < 6.5, so take S/N = 5.5 and $W_{km/s}$ = 100, this gives

$$\left(\frac{M_{HI}}{M_\odot}\right)_{min} = 1.22\times10^4 \left(14.3\right)^2 \sqrt{\frac{100}{200}}\left(5.5\right) = 9.65\times10^6 M_\odot \tag{12}$$

ALFALFA OBSERVATIONS:

For ALFALFA, α.I includes a factor f_β to quantify the fraction of the source flux detected by the beam. For a point source, f_β = 1 and for resolved sources, $f_\beta \approx \Omega_{beam}/\Omega_{source}$. The two polarizations adding together still doubles t_s, but there is no on-off switching (the signal and noise are observed for the full time $\Rightarrow$ no $\frac{1}{2}S_{peak}$ as for LBW). So Equation (4) is written a bit differently,

$$\frac{S}{N} = \frac{f_\beta S_{peak}}{S_{rms}} = f_\beta \left(\frac{M_{HI}}{M_\odot}\frac{1}{2.356\times10^5 D_{Mpc}^2 W_{km/s}}\right)\left(\frac{\sqrt{2\Delta f_{ch} t_s f_t}}{\left(T_{sys}/G\right)}\right)$$

$$\frac{S}{N} = \frac{M_{HI}}{M_\odot}\frac{f_\beta\sqrt{\Delta f_{ch} t_s f_t}}{1.67\times10^5 D_{Mpc}^2 W_{km/s}\left(T_{sys}/G\right)}. \tag{13}$$

They also take the average T/G = SEFD = 3.25 Jy and $f_t = f_{switch} f_{other} f_{smo} = 0.7 f_{smo}$. Re-writing equation (7),

$$f_t = 0.7 f_{smo} = \frac{0.7 W_{km/s}}{2c}\frac{f_0}{\Delta f_{ch}} = \frac{0.35 W_{km/s}}{c}\frac{f_0}{\Delta f_{ch}} \tag{14}$$

giving (since Δf_{ch} cancels out),

$$\frac{S}{N} = \frac{M_{HI}}{M_\odot}\frac{f_\beta\sqrt{\left(0.35\right) t_s f_0}}{1.67\times10^5 D_{Mpc}^2 \left(T_{sys}/G\right)\sqrt{W_{km/s} c}}.$$

$$\left(\frac{S}{N}\right)_{ALFALFA} = \frac{M_{HI}}{M_\odot}\frac{f_\beta\sqrt{\left(0.35\right) t_s \left(1420\times10^6\right)}}{1.67\times10^5 D_{Mpc}^2 \left(3.25\right)\sqrt{W_{km/s}\left(3\times10^5\right)}}.$$

$$\left(\frac{S}{N}\right)_{ALFALFA} = 7.52\times10^{-5}\frac{M_{HI}}{M_\odot}\frac{f_\beta\sqrt{t_s}}{D_{Mpc}^2\sqrt{W_{km/s}}}. \tag{15}$$

The spectral smoothing still increases the S/N up to a maximum of $W_{km/s}$ = 200, so

$$\frac{1}{\sqrt{W_{km/s}}} = \frac{1}{\sqrt{200}}\left(\frac{200}{W_{km/s}}\right)^{-\gamma}$$

where γ = -1/2 for $W_{km/s} < 200$ and γ = -1 for $W_{km/s} \geq 200$. This now gives,

$$\left(\frac{S}{N}\right)_{ALFALFA} = 5.32\times10^{-6}\,\frac{M_{HI}}{M_\odot}\frac{f_\beta\sqrt{t_s}}{D_{Mpc}^2}\left(\frac{W_{km/s}}{200}\right)^{\gamma}.$$

To get it in the form of α.I (4), multiply and divide by 10^6,

$$\left(\frac{S}{N}\right)_{ALFALFA} = 5.32\left(\frac{M_{HI}}{10^6 M_\odot}\right)\frac{f_\beta\sqrt{t_s}}{D_{Mpc}^2}\left(\frac{W_{km/s}}{200}\right)^{\gamma}. \qquad (16)$$

Which is close to α.I's Equation 4:

$$12.3 f_\beta t_s^{1/2}\left(\frac{M_{\rm H\,I}}{10^6\,M_\odot}\right)D_{\rm Mpc}^{-2}\left(\frac{W_{\rm km\,s^{-1}}}{200}\right)^{\gamma} > 6, \qquad (4)$$

It turns out that Mike Jones couldn't get that 12.3 constant, either. Riccardo could not locate the notes where he calculated it, so we don't understand the difference. One idea is that they did something slightly different with f_{smo} as I did in Equation (14), following Mike's work for LBW.